REALNE F-JE JEDNE REALNE PROMJ.

Pojam funkcije

A, B - dva neprazna skupa i $x \in A$, a f neko pravilo (zakon, postupak) kojim se svakom elementu x pridružuje samo jedan el. $y = f(x)$ iz skupa B.
(TAČNO)

Tada kažemo da je f preslikavanje ili (funkcija) skupa A u skup B.

$$f : A \to B \quad \text{ili} \quad A \xrightarrow{f} B$$

x - argument ili nezav. promj.
y - funkcija ili zavisno promj.

A - oblast definisanosti (DOMEN) f je $\quad A = D(f) = \{x \in R \mid (\exists y \in R)\ y = f(x)\}$

B - skup vrijednosti (KODOMEN) fje $\quad B = R(f) = \{y \in R \mid (\exists x \in D)\ y = f(x)\}$

Ako su $A, B \subseteq R$ onda je fja f naziva REALNA FUNKCIJA.

Npr: $y = 2x + 5$, $\quad y = \sqrt{x}$, $\quad y = \frac{x}{x-1}$

$D: (-\infty, \infty)$, $\quad [0, +\infty)$, $\quad (-\infty, 1) \cup (1, +\infty)$

PODJELA FUNKCIJA:

- FUNKCIJE
  - ALGEBARSKE
    - RACIONALNE
      - CIJELE (POLINOMI)
      - RAZLOMLJENE
        - PRAVE
        - NEPRAVE
    - IRACIONALNE
  - TRANSCEDENTNE (trig., logaritamske, eksponenc., ciklom. - sve koje nisu alg.)

Primjer algebarskih fja: $y = \sqrt{x} + 2$ (IRACION.), $\quad y = \frac{x+2}{x-1}$ (RACION. (RAZLOMLJENA)), $\quad y = \frac{x^3 + x + 1}{x^2 - 1}$ (—"— NEPRAVA), $\quad y = x^4 - 3x^3 + 2$ (CIJELA)

Razlomljena racionalna algebarska f-ja:

$$f(x) = \frac{P_m(x)}{P_n(x)} \qquad \left(\begin{array}{l} m < n, \text{ PRAVA} \\ m \geq n, \text{ NEPRAVA} \end{array} \right)$$

Način zadavanja f-je

a) ANALITIČKI

1° EKSPLICITNI OBLIK $\quad y = f(x)$

$$y = \sqrt{x^2 - 4x}, \qquad y = \frac{x^2-1}{x^3+2}, \qquad y = 2x+5$$

2° IMPLICITNI OBLIK $\quad F(x,y) = 0$

$$2x - y + 5 = 0, \qquad x^2 + y^2 - r^2 = 0$$

3° SA DVA ILI VIŠE IZRAZA

$$y = \begin{cases} x+3, & x < 0 \\ 3, & 0 \leq x < 5 \\ x-2, & x \geq 5 \end{cases}$$

4° PARAMETARSKI OBLIK

$$\begin{array}{l} x = \varphi(t) \\ y = \psi(t) \end{array} \qquad t \in [t_1, t_2] \subset R$$

b) TABLIČNI

c) GRAFIČKI

Neke osobine funkcija

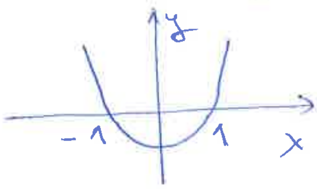
Def 1. Skup svih vrijednosti x iz oblasti def. f-je $y = f(x)$ za koje je $f(x) = 0$ su nule funkcije.

Npr.

$$f(x) = x^2 - 1$$

$$x^2 = 1$$

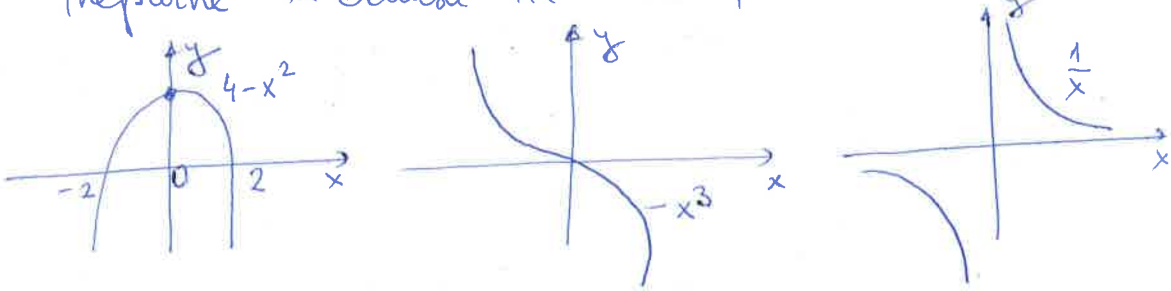
$$x = -1, \quad x = +1$$



Grafik siječe ili dodiruje x-osu

Def 2 Za fju $f(x)$ kažemo da je PARNA ako za svako x iz domene važi $f(-x) = f(x)$, a da je NEPARNA ako za svako x iz $D(f)$ važi $f(-x) = -f(x)$.

Grafik parne fje je simetričan u odnosu na y-osu, a neparne u odnosu na koord. početak.



Def 3: (monotonost fje):

Fja je RASTUĆA u int. (a,b) ako za svako $x_1, x_2 \in (a,b)$

važi $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$

OPADAJUĆA : $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$

NEOPADAJUĆA : $x_1 < x_2 \Rightarrow f(x_1) \leq f(x_2)$

NERASTUĆA : $x_1 < x_2 \Rightarrow f(x_1) \geq f(x_2)$

Def 4: (ograničenost): Fja $f(x)$ je u intervalu (a,b):

a) ograničena s gornje strane, ako postoji realan broj M takav da je za sve $x \in (a,b)$: $f(x) \leq M$

b) ograničena s donje strane, — " — m — " —

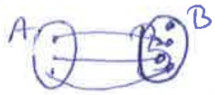
— " — $: m \leq f(x)$

c) OGRANIČENA, ako je ograničena i s gornje i s donje strane tj. postoje brojevi m i $M \in R$, $m \leq f(x) \leq M$ za sve $x \in (a,b)$

Def 5: Fja $f: D_f \to R$ je periodična ako postoji realan broj $\omega \neq 0$ takav da $x+\omega \in D_f$ i $(\forall x \in D_f)$ $f(x+\omega) = f(x)$ ω - PERIOD F-JE

NAJMANJI TAKAV POZ. BR. T — OSNOVNI PERIOD F-JE

Za presl. $f: A \to B$ kod kojeg se različitim elem. iz A pridružuju razl. el. iz B kažemo da je injektivno (injekcija) tj. jedan-jedan presl. „1-1" $(\forall x_1, x_2 \in A)\ f(x_1) = f(x_2) \Rightarrow x_1 = x_2$



Za presl. $f: A \to B$ kod kojeg svaki el. iz B ima svoj original u A kažemo da je surjektivno tj. (SIRJEKCIJA) „NA" preslikavanje $(B = \mathcal{R}(f))$. $(\forall y \in B)(\exists x \in A)\ f(x) = y$.

Presl. $f: A \to B$ koje je „1-1" i „NA" je bijekcija - obostrano jednoznačno preslikavanje. $(\forall y \in B)(\exists! x \in A)\ f(x) = y$

Tada postoji inverzna fja (presl.) $f^{-1}: B \to A$.

$$(\forall x \in D(f) = A)\ (f^{-1}(f(x)) = x) \qquad f \circ f^{-1} = id_R(f)$$

$$(\forall y \in \mathcal{R}(f) = B)\ (f(f^{-1}(y)) = y) \qquad f^{-1} \circ f = id_D(f)$$

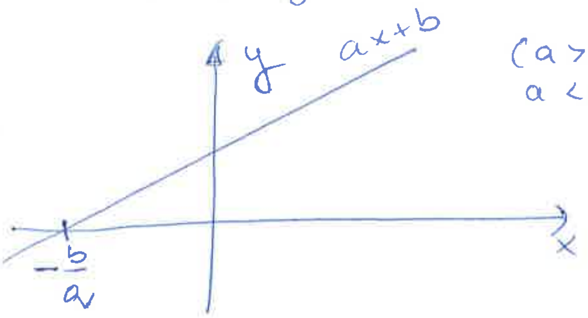
Grafik fje $f(x)$ je simetričan sa grafikom fje $f^{-1}(x)$ u odnosu na pravu $y = x$. Prema tome ako je grafik fje simetričan u odnosu na pravu $y = x$ tada je fja sama sebi inverzna.

ELEMENTARNE FUNKCIJE

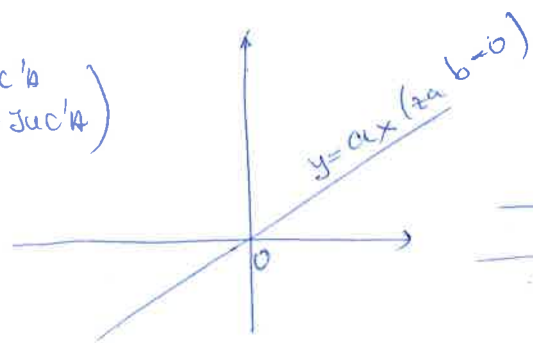
Linearna funkcija

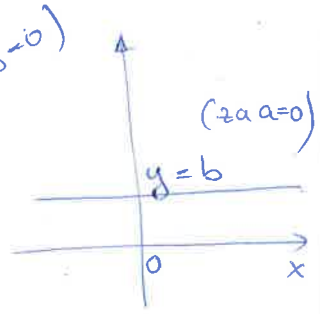
$$f(x) = ax + b \quad , a \neq 0 \quad (a, b \in \mathbb{R})$$

Nula fje: $x = -\frac{b}{a}$



(a > 0 RASTUĆA
a < 0 OPADAJUĆA)





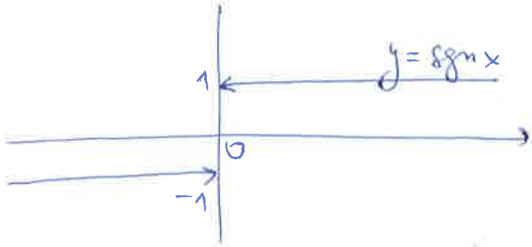
Apsolutna vrijednost

$y = |x|$ $\qquad |x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$

f-ja je PARNA

Funkcija znaka

$y = \operatorname{sgn} x$, $\operatorname{sgn} x = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$



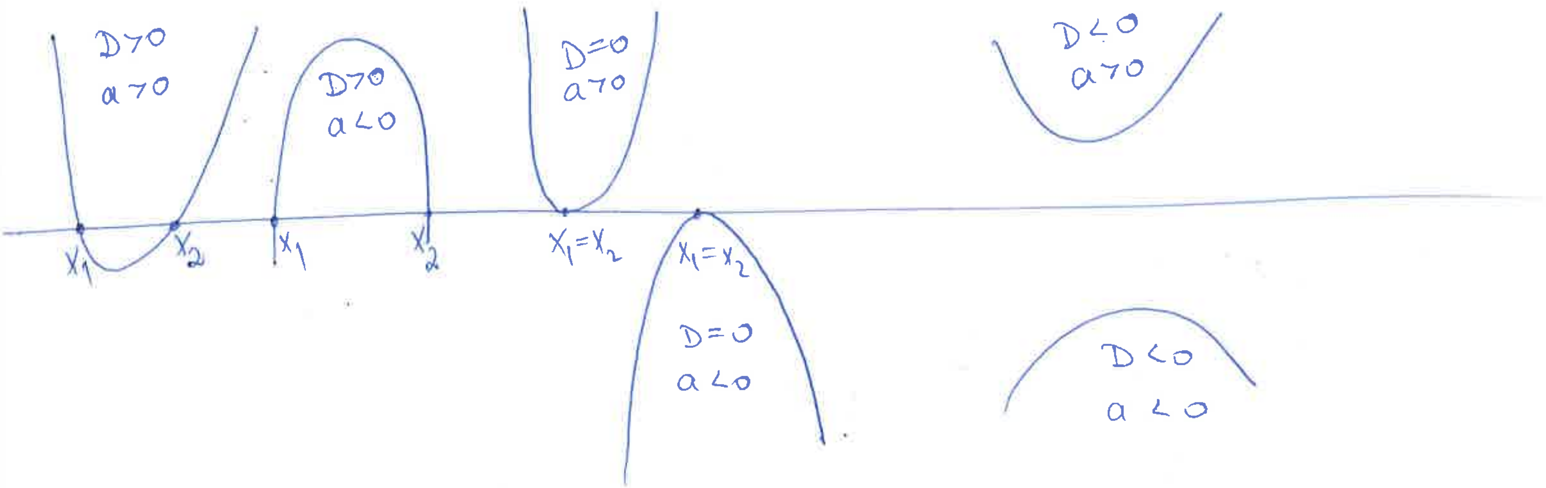
f-ja je neparna

Kvadratna funkcija

$y = ax^2 + bx + c$, $a \neq 0$ $(a, b, c \in \mathbb{R})$

$D = b^2 - 4ac$ DISKRIMINANTA

$T\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$ 2a $a > 0$ - MIN. u T
$a < 0$ - MAX. u T



$f(x) = a(x - x_1)(x - x_2)$

Stepena f-ja

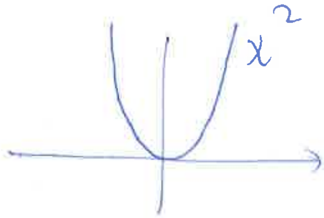
$$y = x^n \qquad (n \in \mathbb{Z})$$

$x=0$ – nula f-je

1° $n>0$

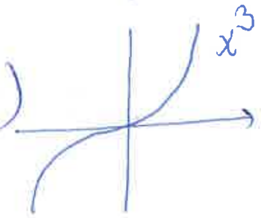
a) n – paran broj, $n = 2k$ $(k \in \mathbb{N})$

F-ja je PARNA



b) n – neparan broj, $n = 2k+1$ $(k \in \mathbb{N})$

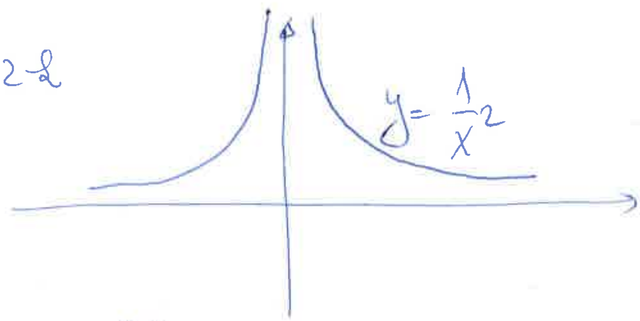
NEPARNA



2° $n<0$

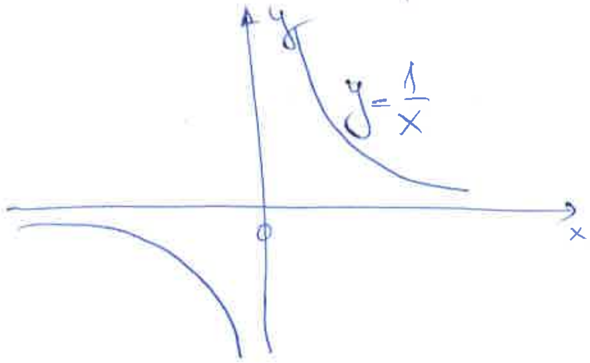
a) n – paran broj, $n = 2k$

PARNA



b) n – neparan broj

NEPARNA



Funkcija KORIJEN

$$y = \sqrt[n]{x} \qquad (n \in \mathbb{N})$$

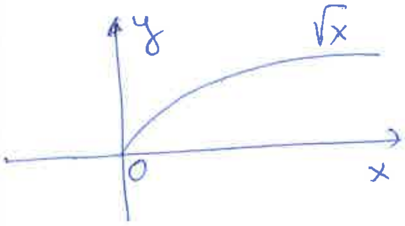
n – paran broj def. je za $x \geq 0$

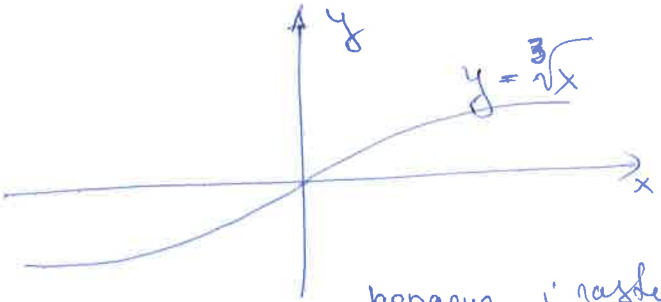
n – neparan broj — " — svako $x \in \mathbb{R}$

Nula: $x = 0$

$n=2$ i $n=3$

$y=\sqrt{x}$, $y=\sqrt[3]{x}$





nenegativna i rastuća

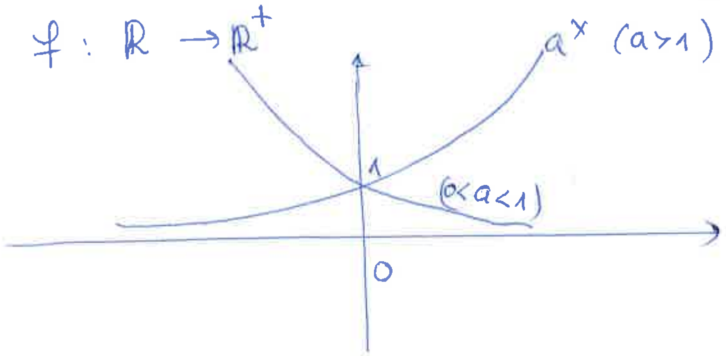
neparna i rastuća

Eksponencijalna f-ja

$$y=a^x \quad (a>0,\ a\neq 1)$$

$D=\mathbb{R}$, nema realnih nula, uvijek pozitivna.

$f:\mathbb{R}\to\mathbb{R}^+$



OSNOVNE OSOBINE:

1) $a^{x+y}=a^x\cdot a^y$
2) $a^{x-y}=\dfrac{a^x}{a^y}$
3) $(a^x)^y=a^{x\cdot y}$
4) $(a\cdot b)^x=a^x\cdot b^x$
5) $\left(\dfrac{a}{b}\right)^x=\dfrac{a^x}{b^x}$

Spec. za $a=e$: $f(x)=e^x$

Logaritamska f-ja

Inverzna od eksponencijalne

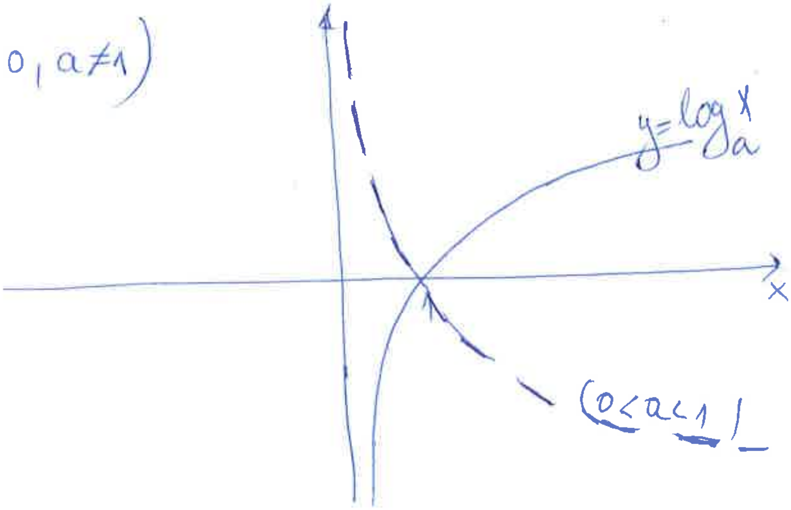
$$y=\log_a x \quad (a>0,\ a\neq 1)$$

$D: x>0$, $N: x=1$

$a>1$ – RASTUĆA

$0<a<1$ – OPADAJUĆA

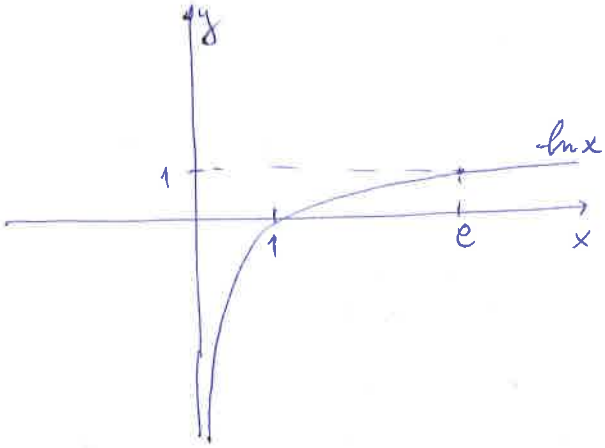
$f:\mathbb{R}^+\to\mathbb{R}$



$f(x) = \ln x$ PRIRODNI LOGARITAM

$\ln x = \log_e x$

$e \approx 2{,}718$



OSOBINE LOGARITAMA:

1) $\log_a 1 = 0$

2) $\log_a a = 1$; $a^{\log_a x} = x$

3) $\log_a x \cdot y = \log_a x + \log_a y$; $\log_a \frac{x}{y} = \log_a x - \log_a y$

4) $\log_a x^b = b \cdot \log_a x$; $\log_{a^c} x = \frac{1}{c} \log_a x$

5) $\log_b a = \dfrac{1}{\log_a b}$

6) $\log_b a = \dfrac{\log_c a}{\log_c b}$

$$\boxed{y = \log_a x \iff x = a^y}$$

PRIMJERI :

1 Skicirati graf fje:

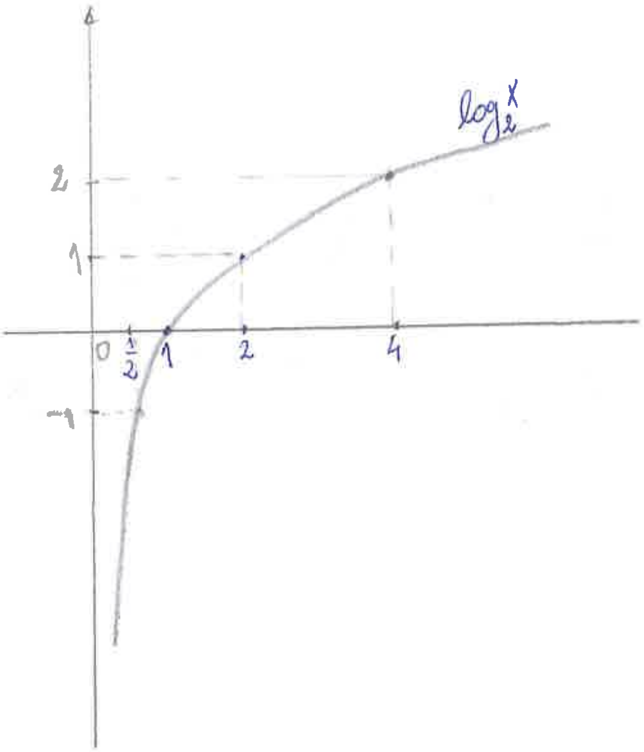
a) $y = \log_2 x$ $\qquad D_f: x \in (0, +\infty)$

Za $x = 1 \Rightarrow y = \log_2 1 = 0$

Za $x = 2 \Rightarrow y = \log_2 2 = 1$

Za $x = 4 \Rightarrow y = \log_2 4 = 2$ jer je $2^2 = 4$

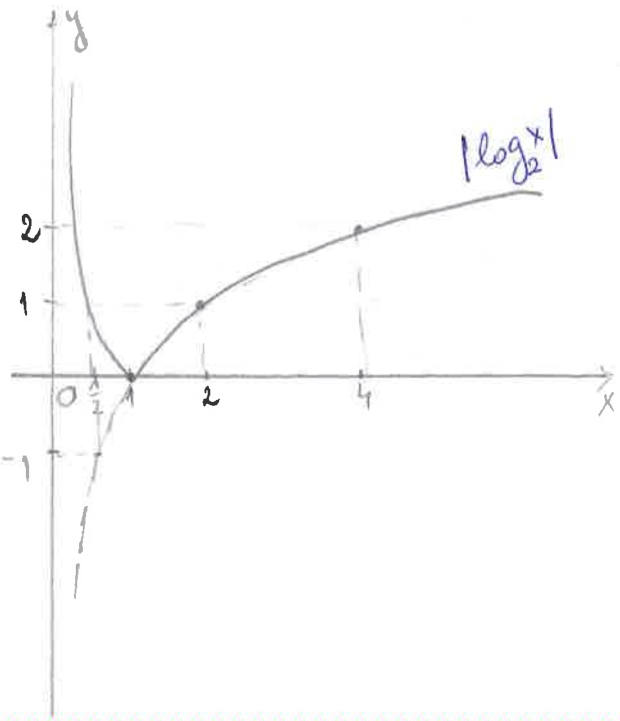
Za $x = \frac{1}{2} \Rightarrow y = \log_2 \frac{1}{2} = \log_2 2^{-1} = -1 \cdot \log_2 2 = -1$



x	1	2	4	$\frac{1}{2}$
y	0	1	2	-1

b) $y = |\log_2 x|$

Iskoristićemo graf fje iz a)



2) Odrediti domen, nule i skicirati grafik funkcije:

$$y = \log_3 (x+1)$$

RJ. DOMEN: $x+1 > 0$

$x > -1$

$D_f: \ x \in (-1, +\infty)$

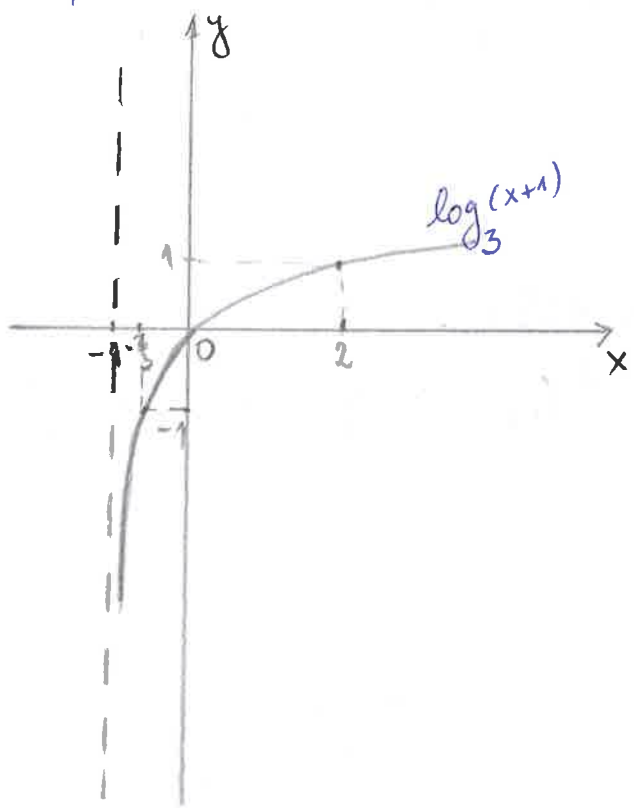
VERTIKALNA ASIMPTOTA JE: $x = -1$

NULE: $\log_3 (x+1) = 0$

$x+1 = 1$

$\boxed{x = 0}$ NULA FUNKCIJE

GRAFIK:



x	0	2	$-\frac{2}{3}$
y	0	1	-1

Za $x = 2$: $y = \log_3 3 = 1$

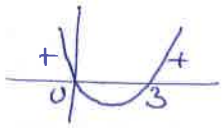
Za $x = -\frac{2}{3}$: $y = \log_3 \frac{1}{3} = -1$

3) Odrediti domen i nule funkcije:

$$y = \log_2 (x^2 - 3x)$$

RJ: Domen: $x^2 - 3x > 0$

$x(x-3) > 0$ - KVADRATNA NEJEDNAČINA



D_f: $x \in (-\infty, 0) \cup (3, +\infty)$

Nule: $x^2 - 3x = 1$ (logaritam ima nule u jedinici)

$x^2 - 3x - 1 = 0$

$$x_{1,2} = \frac{3 \pm \sqrt{9+4}}{2} = \frac{3 \pm \sqrt{13}}{2}$$

Obe nule pripadaju D_f.

Logaritamske jednačine

$$\log_a f(x) = \log_a g(x) \iff \left(f(x) = g(x) \wedge f(x) > 0 \wedge g(x) > 0 \right)$$

1) Riješiti jednačine:

a) $\log_2 (2x+3) = \log_2 4$ USLOV: $2x + 3 > 0$

$x > -\frac{3}{2}$

$2x + 3 = 4$

$2x = 1$

$x = \frac{1}{2}$ → RJEŠENJE ZADOVOLJAVA USLOV.

b) $\log_2 8 + \log_{\frac{1}{2}}(x+1) = 5$

USLOV:
$x+1>0$
$\underline{x>-1}$

$\log_2 2^3 + \log_{2^{-1}}(x+1) = 5$

$3 - \log_2(x+1) = 5$

$-\log_2(x+1) = 2$

$\log_2(x+1) = -2$

$x+1 = 2^{-2}$

$x = \frac{1}{4} - 1$

$\underline{x = -\frac{3}{4}}$ – RJ. zadovoljava uslov da je $x>-1$

c) $\log(5-x) + 2\log\sqrt{3-x} = 1$

USLOV:
$5-x>0 \wedge 3-x>0$
$x<5 \wedge x<3$
$\underline{x \in (-\infty, 3)}$

$\log(5-x) + 2\log(3-x)^{\frac{1}{2}} = \log 10$

$\log(5-x) + \cancel{2}\cdot\frac{1}{\cancel{2}}\log(3-x) = \log 10$

$\log(5-x)(3-x) = \log 10$

$(5-x)(3-x) = 10$

$15 - 8x + x^2 = 10$

$x^2 - 8x + 5 = 0$

$x_{1,2} = 4 \pm \sqrt{11}$

$x_1 = 4 - \sqrt{11}$ zadovoljava uslov definisanosti jednačine

$x_2 = 4 + \sqrt{11}$ ne zadovoljava uslov, pa nije rješenje jednačine

(2.) Riješiti jednačinu:

$$\log_2 x + \log_x 2 = \frac{5}{2} \quad , \qquad \text{USLOV: } x > 0 \ , \ x \neq 1$$

RJ. $\log_2 x + \dfrac{1}{\log_2 x} = \dfrac{5}{2}$, UVESTI SMJENU: $\log_2 x = t$

$$t + \frac{1}{t} = \frac{5}{2} \quad / \cdot 2t$$

$$2t^2 + 2 = 5t$$

$$2t^2 - 5t + 2 = 0$$

$$t_2 = \frac{5 \pm \sqrt{25-16}}{4} = \frac{5 \pm 3}{4}$$

$$t_1 = \frac{1}{2} \Rightarrow \log_2 x = \frac{1}{2} \Rightarrow \underline{x_1 = 2^{\frac{1}{2}} = \sqrt{2}}$$

$$t_2 = 2 \Rightarrow \log_2 x = 2 \Rightarrow \underline{x_2 = 2^2 = 4}$$

(3) Odrediti domen i nule funkcije:

$$y = \ln \frac{x^2-4}{2x-1}$$

RJ. DOMEN: $\dfrac{x^2-4}{2x-1} > 0$

	$-\infty$ … -2	-2 … $\frac{1}{2}$	$\frac{1}{2}$ … 2	2 … $+\infty$
x^2-4	+	−	−	+
$2x-1$	−	−	+	+
K	−	+	−	+

$$D_f: \ x \in \left(-2, \frac{1}{2}\right) \cup (2, +\infty)$$

NULE: $\dfrac{x^2-4}{2x-1} = 1$

$$x^2 - 4 = 2x - 1$$

$$x^2 - 2x - 3 = 0$$

$$\underline{x_1 = -1}, \quad \underline{x_2 = 3} \quad \in D_f$$

TRIGONOMETRIJSKE F-JE

$y = \sin x$, $y = \cos x$, $y = tg\,x$ i $y = ctg\,x$

1. $y = \sin x$

PERIODIČNA sa osnovnim periodom $T = 2\pi$

NULE F-JE: $x = l\pi$, $l = 0, \pm 1, \pm 2, \ldots$

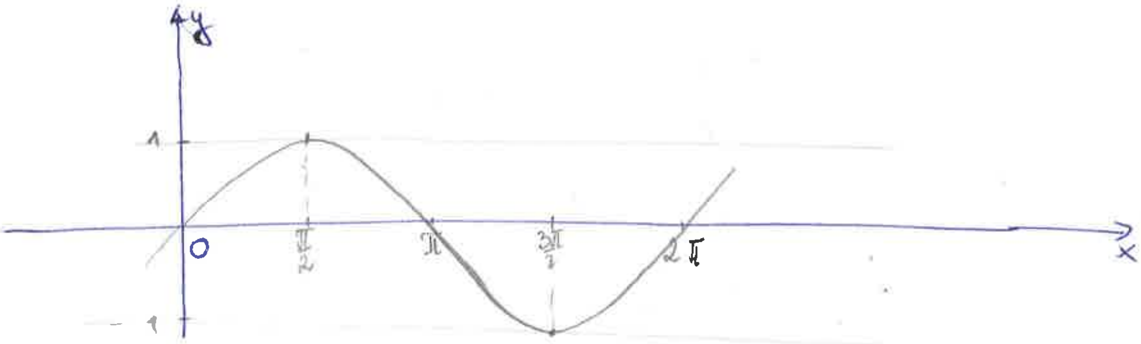
OGRANIČENA $(-1 \leq \sin x \leq 1)$

F-JA JE NEPARNA

EKSTREMNE VRIJEDNOSTI: Maksimume za $x = (4l+1)\frac{\pi}{2}$ $(l \in Z)$

Minimume za $x = (4l-1)\frac{\pi}{2}$ $(l \in Z)$

Sinusoida $y = \sin x$ $\sin x : R \to [-1, 1]$

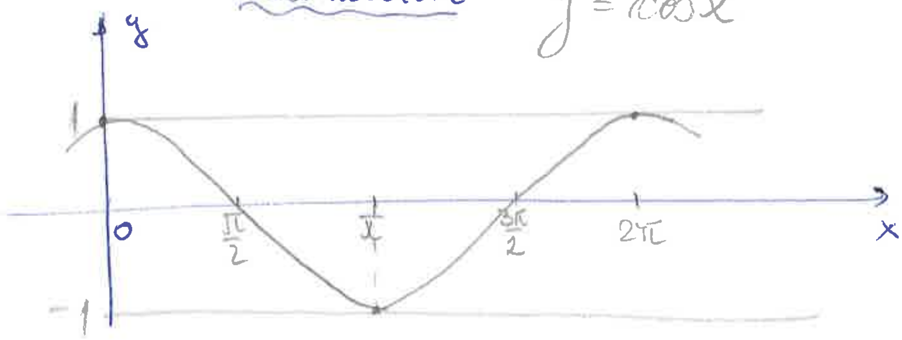


2. $y = \cos x$

$T = 2\pi$, Nule: $x = (2l+1)\frac{\pi}{2}$, $(-1 \leq \cos x \leq 1)$ PARNA

Max: za $x = 2l\pi$, MIN: za $x = (2l+1)\pi$

Kosinusoida $y = \cos x$ $\cos x : R \to [-1, 1]$



3. $y = tg\, x \qquad tg\, x = \dfrac{\sin x}{\cos x}$

$D_f: \; x \neq (2k+1)\frac{\pi}{2} \quad (k \in \mathbb{Z})$, $T = \pi$ – OSNOVNI PERIOD

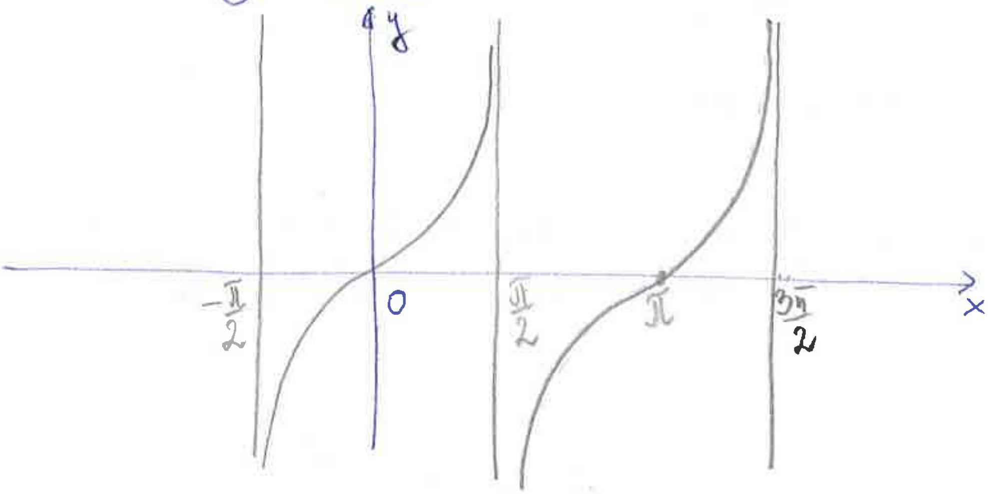
NULE: $x = k\pi \quad (k \in \mathbb{Z})$

VERTIKALNE ASIMPTOTE: $x = (2k+1)\frac{\pi}{2} \quad (k \in \mathbb{Z})$

NEPARNA

tangensoida

$tg\, x : \mathbb{R} \setminus \{(2k+1)\frac{\pi}{2}\} \to \mathbb{R}$

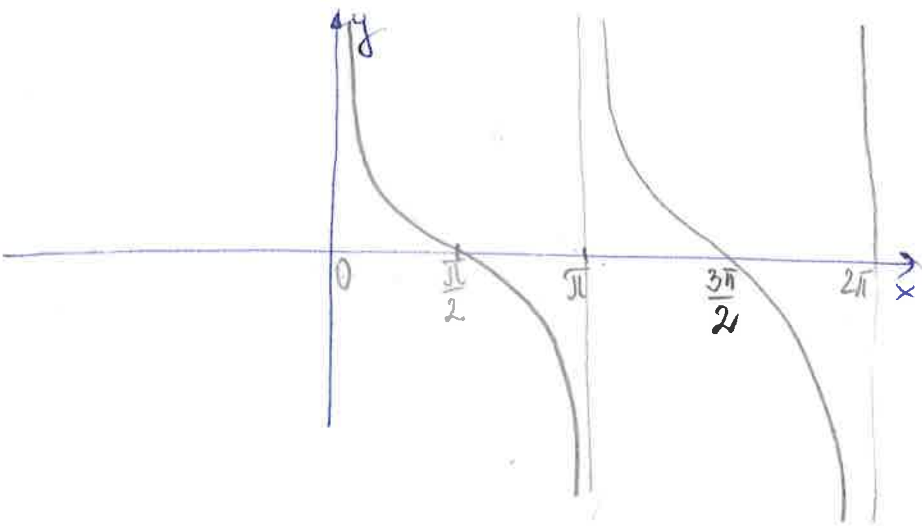


4. $y = ctg\, x, \qquad ctg\, x = \dfrac{\cos x}{\sin x}, \qquad ctg\, x : \mathbb{R} \setminus \{k\pi\} \to \mathbb{R}$

$D_f: \; x \neq k\pi \quad (k \in \mathbb{Z})$, $T = \pi$, NULE: $x = (2k+1)\frac{\pi}{2}$

NEPARNA, VERT. A. $x = k\pi \quad (k \in \mathbb{Z})$

kotangensoida



OSNOVNE TRIGONOMETRIJSKE IDENTIČNOSTI

1° $\sin^2\alpha + \cos^2\alpha = 1$

2° $\sin 2\alpha = 2\sin\alpha\cos\alpha$

3° $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

4° $\sin(\alpha \pm \beta) = \sin\alpha \cdot \cos\beta \pm \cos\alpha \cdot \sin\beta$

5° $\cos(\alpha \pm \beta) = \cos\alpha \cdot \cos\beta \mp \sin\alpha \sin\beta$

...

1. Riješiti jednačinu:

$$\sin x + \cos^2 x = 1$$

Rj.

$$\sin x + \cancel{1} - \sin^2 x = \cancel{1}$$

$$\sin x - \sin^2 x = 0$$

$$\sin x (1 - \sin x) = 0$$

$$\sin x = 0 \quad \vee \quad \sin x = 1$$

$$x = k\pi \ (k \in \mathbb{Z}) \quad , \quad x = \frac{\pi}{2} + 2k\pi \ (k \in \mathbb{Z})$$

2. $\cos 2x = \cos x$

$$\cos^2 x - \sin^2 x = \cos x$$

$$\cos^2 x - (1 - \cos^2 x) = \cos x$$

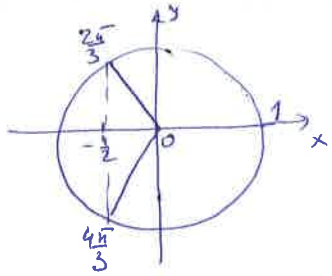
$$2\cos^2 x - \cos x - 1 = 0, \qquad \text{SMJENA : } \cos x = t$$

$$2t^2 - t - 1 = 0$$

$$t_1 = -\frac{1}{2}, \quad t_2 = 1$$

VRATIMO SMJENU:

$$\cos x = -\frac{1}{2}$$



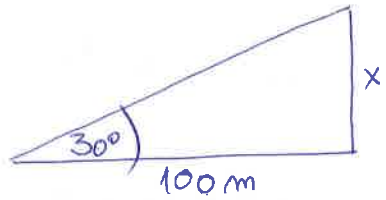
$$x_1 = \frac{2\pi}{3} + 2k\pi \quad (k \in \mathbb{Z})$$

$$x_2 = \frac{4\pi}{3} + 2k\pi$$

$$\cos x = 1$$

$$x_3 = 2k\pi$$

(3) Odrediti visinu nebodera koji se sa udaljenosti od 100m vidi pod uglom od 30°.



RJ. $$\operatorname{tg} 30° = \frac{x}{100}$$

$$\Rightarrow \quad x = 100 \cdot \operatorname{tg} 30°$$

$$x = 100 \cdot \frac{\sqrt{3}}{3}$$

$$x = \frac{100\sqrt{3}}{3}\,m$$

(4) Ako je $\sin\alpha = \frac{5}{13}$ i $\sin\beta = \frac{12}{13}$ (α i β oštri uglovi) izračunati $\sin(\alpha+\beta)$ i $\cos(\alpha-\beta)$. Samostalno!

(5) Izračunati stranicu c trougla u kome je $a=3$, $b=8$ i $\gamma=60°$.

RJ. KOSINUSNA TH:

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

$$c^2 = 9 + 64 - 2\cdot 3\cdot 8\cdot \cos 60°$$

$$c^2 = 49 \quad \Rightarrow \quad \boxed{c = 7}$$

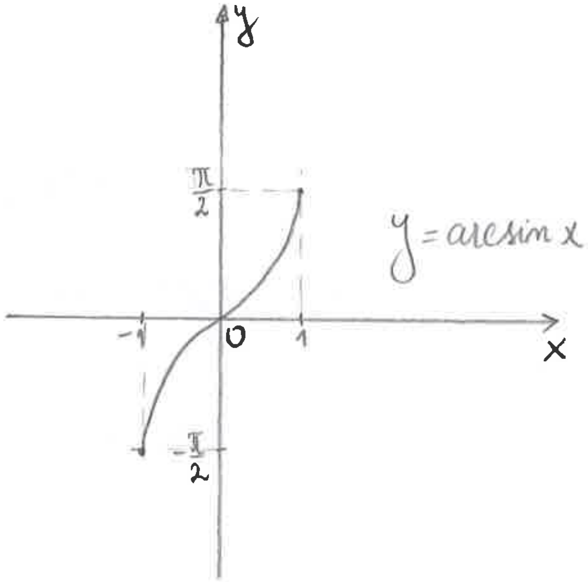
CIKLOMETRIJSKE FUNKCIJE
(ARKUS FUNKCIJE)
(Inverzne trigonometrijske funkcije)

$\arcsin x$, $\arccos x$, $\operatorname{arctg} x$, $\operatorname{arcctg} x$

1° Arkus sinus

F-ja $y = \sin x$ nema inverznu funkciju jer nije bijekcija!
Ako posmatramo njenu restrikciju na intervalu $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ dobijamo arkus sinus funkciju

$$\arcsin x : [-1,1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$



$\arcsin(\sin x) = x$ za $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

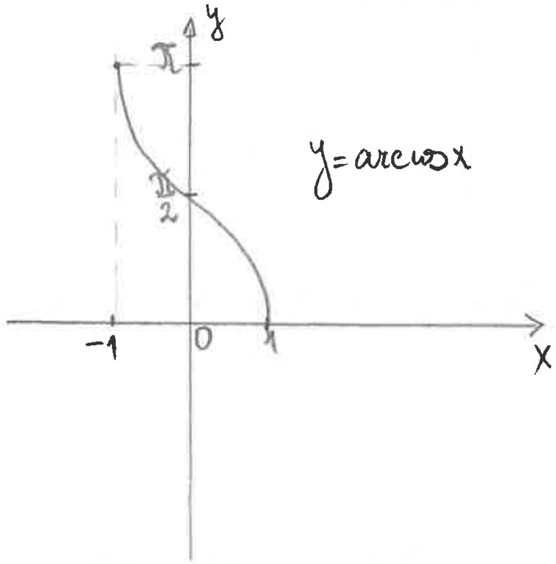
$\sin(\arcsin x) = x$ za $x \in [-1,1]$

$\arcsin(1) = \frac{\pi}{2}$

2° Arkus kosinus

Takođe posmatramo restrikciju f-je $y = \cos x$ na intervalu $[0, \pi]$. (jer ni $\cos x$ nije bijekcija).

$$\arccos x : [-1,1] \to [0, \pi]$$



$\arccos(\cos x) = x$ za $x \in [0, \pi]$

$\cos(\arccos x) = x$ za $x \in [-1, 1]$

<u>Npr.</u>

$\arccos\left(\cos\frac{\pi}{3}\right) = \arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$

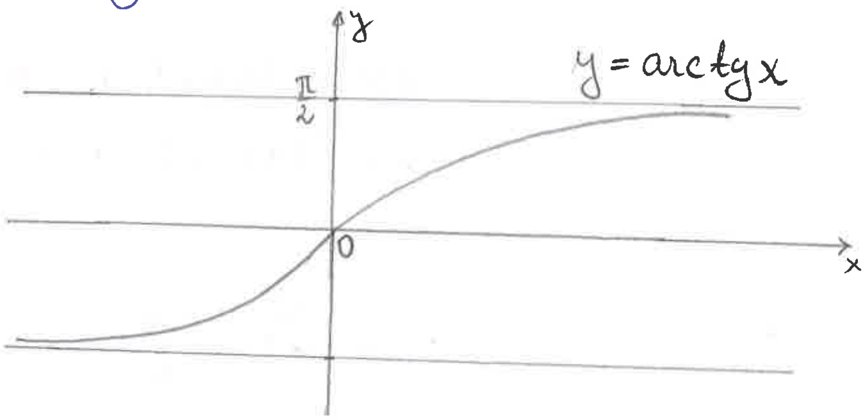
$\arccos(0) = \frac{\pi}{2}$

$\arccos\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

3° Arkus tangens

Restrikcija fje $y = \operatorname{tg} x$ na intervalu $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

$\operatorname{arctg} x : R \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



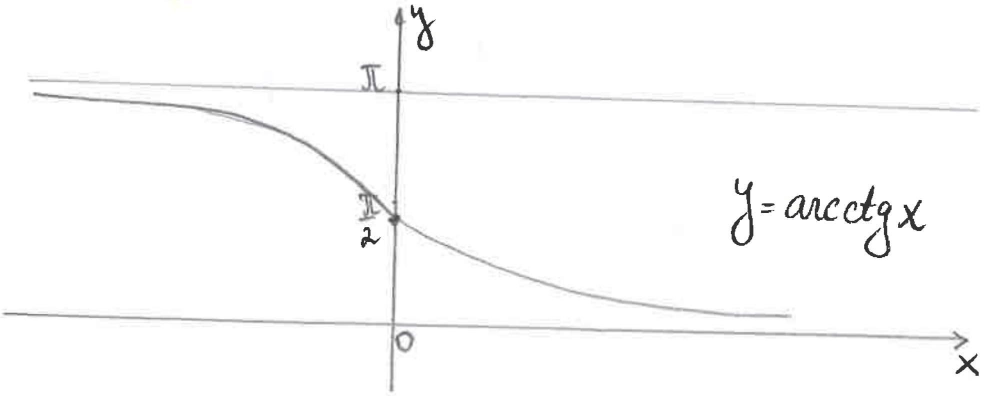
$\operatorname{arctg}(\operatorname{tg} x) = x, \quad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\operatorname{tg}(\operatorname{arctg} x) = x, \quad x \in R$

$\operatorname{arctg}(1) = \frac{\pi}{4}$

4° Arkus kotangens

$\operatorname{arcctg} x : R \to [0, \pi]$

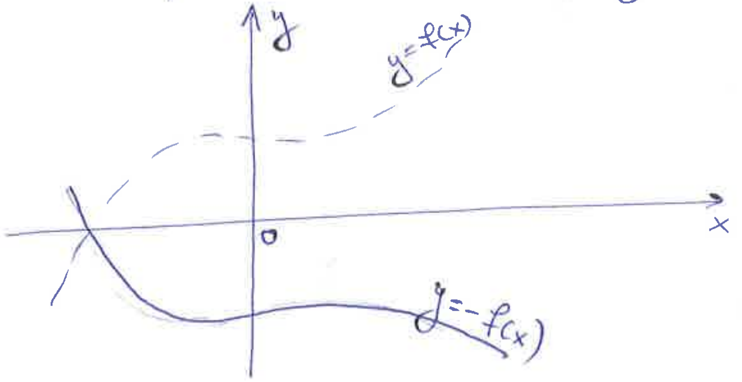


$\operatorname{arcctg}(\operatorname{ctg} x) = x, \quad x \in [0, \pi]$

$\operatorname{ctg}(\operatorname{arcctg} x) = x, \quad x \in R$

GRAFICI ELEMENTARNIH FUNKCIJA

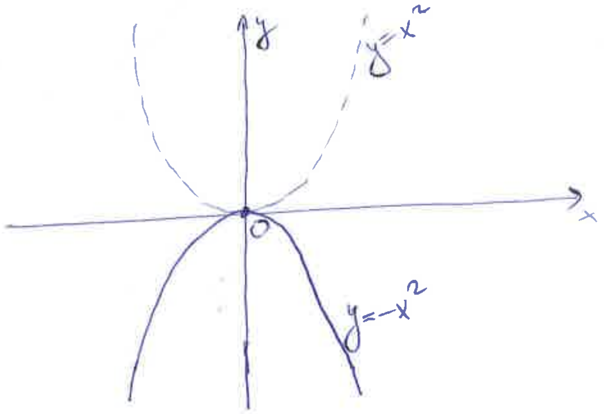
I slučaj Nacrtati sliku krive $y=-f(x)$ ako nam je poznata slika krive $y=f(x)$.



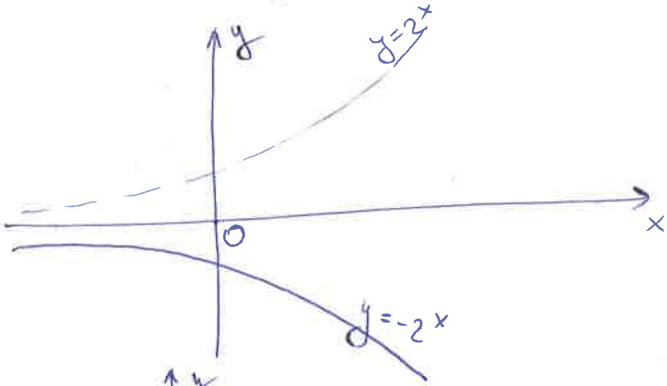
Da bi dobili sliku krive $y=-f(x)$ treba promijeniti znak ordinatama krive $y=f(x)$. Krive $y=-f(x)$ i $y=f(x)$ su simetrične u odnosu na osu Ox.

Primjeri:

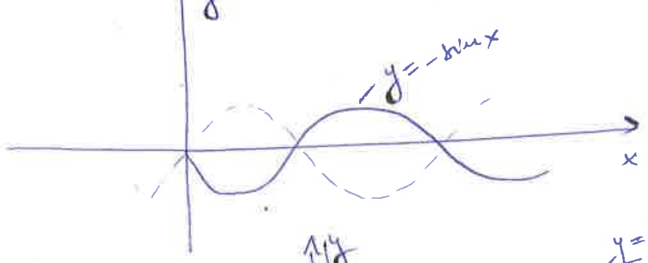
1) $y=-x^2$



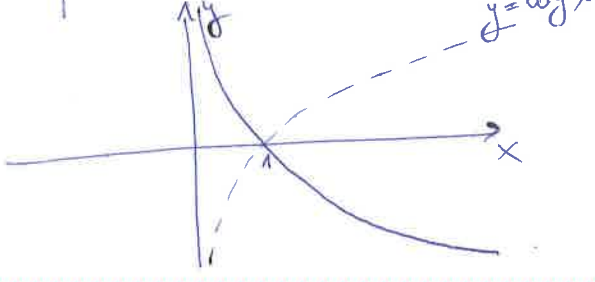
2) $y=-2^x$



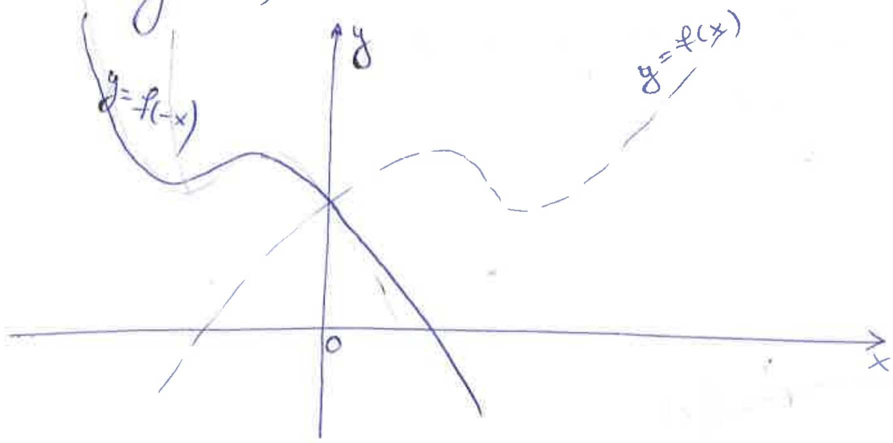
3) $y=-\sin x$



4) $y=\ln\frac{1}{x}=\ln x^{-1}=-\ln x$



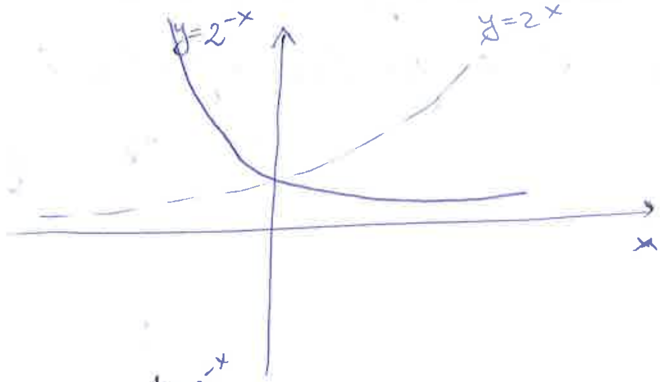
II slučaj Nacrtati sliku krive $y=f(-x)$ ako je poznata slika krive $y=f(x)$



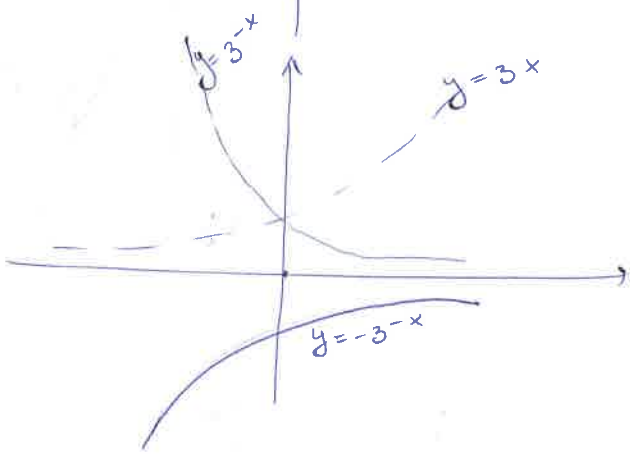
Krive $f(x)$ i $f(-x)$ su simetrične u odnosu na osu Oy.

Primjeri:

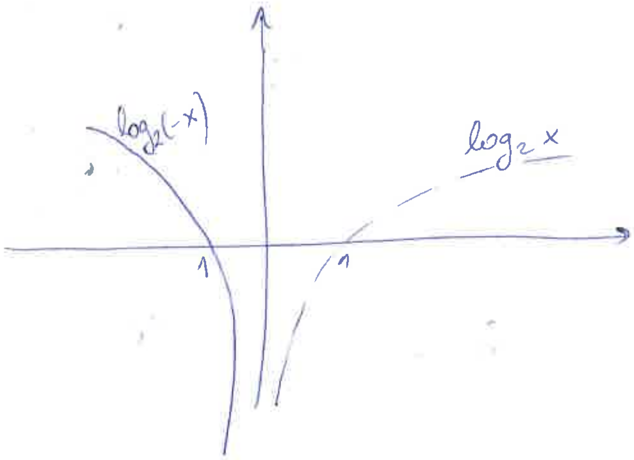
1) $y=2^{-x}$



2) $y=-3^{-x}$



3) $y=\log_2(-x)$

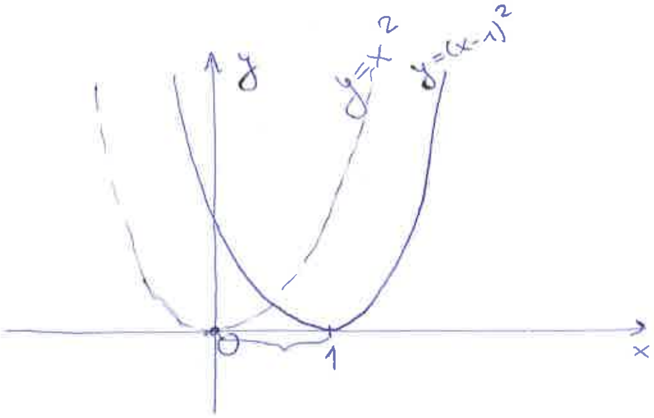


Sliku krive $y=f(x-c)$ dobijamo ako sliku krive $y=f(x)$ pomjerimo za c u pravcu ose Ox.

Primjer:

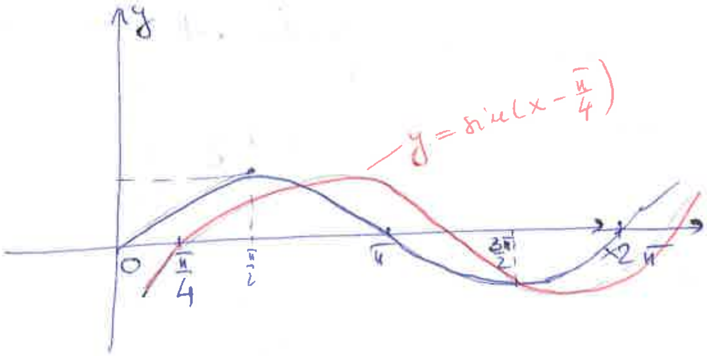
1) $y=(x-1)^2$

$y=x^2$

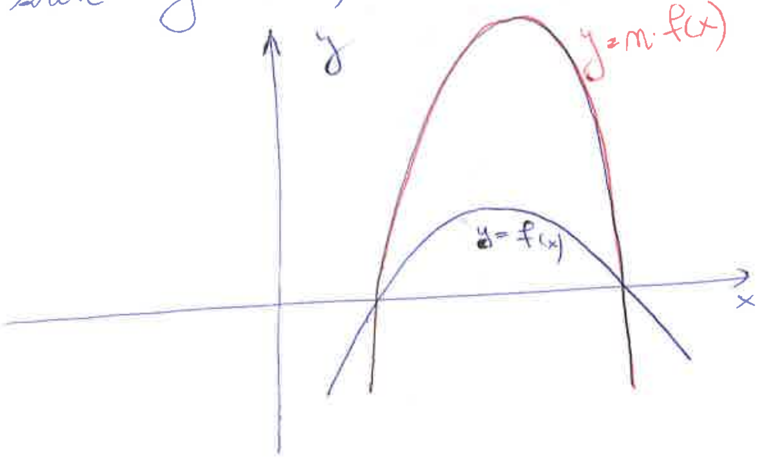


2) $y=\sin\left(x-\frac{\pi}{4}\right)$

$y=\sin x$



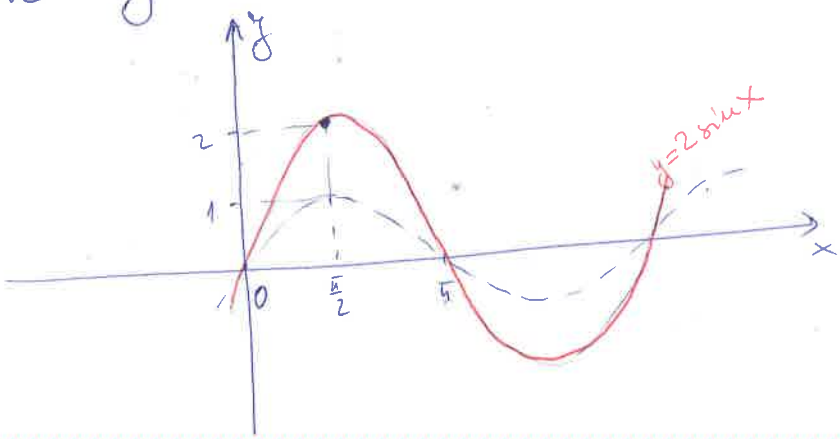
V slučaj Nacrtati sliku krive $y=m\cdot f(x)$ ako se zna slika krive $y=f(x)$ $(m=2)$



Ordinate krive $y=m\cdot f(x)$ su m puta veće od ordinata krive $y=f(x)$.

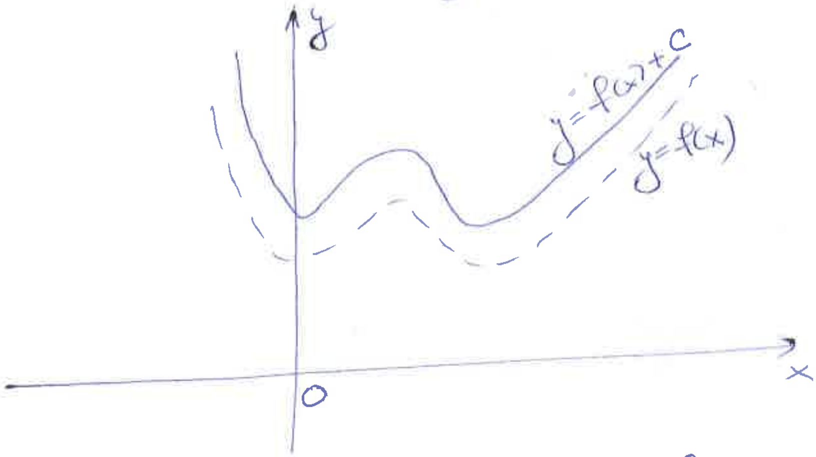
Primjer:

$y=2\cdot\sin x$



III slučaj

Nacrtati sliku krive $y=f(x)+c$ ako je poznata slika krive $y=f(x)$

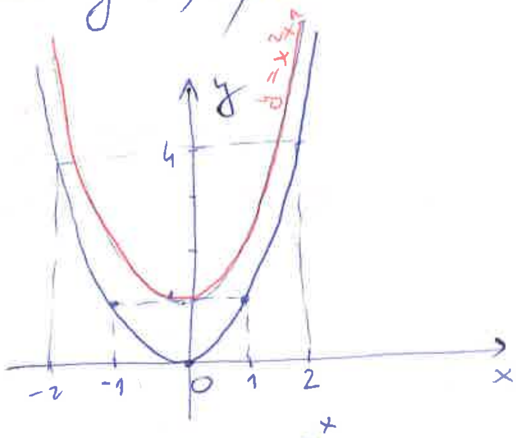


Ako sve ordinate krive $y=f(x)$ povećamo za c dobijemo sliku krive $y=f(x)$.

Primjer:

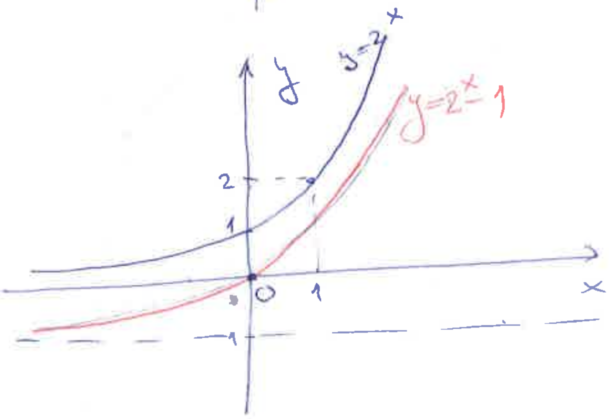
1) $y=x^2+1$

$y=x^2$



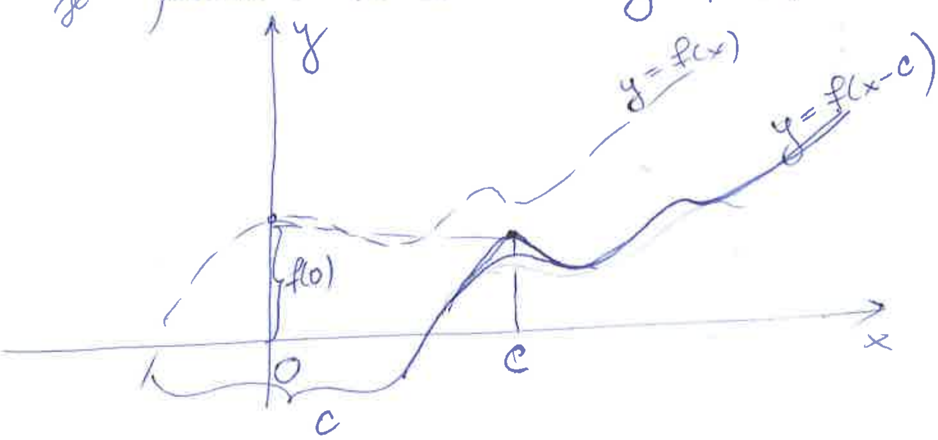
2) $y=2^x-1$

$y=2^x$



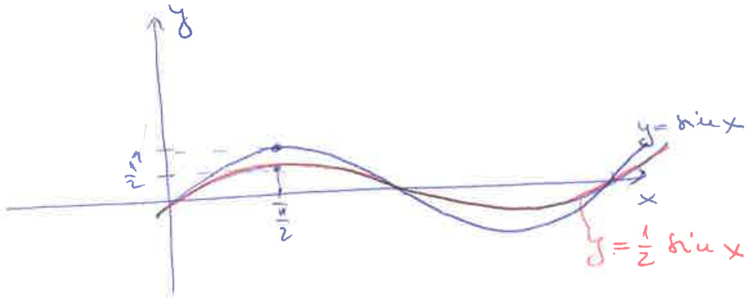
IV slučaj

Nacrtati sliku krive $y=f(x-c)$ ako nam je poznata slika krive $y=f(x)$.



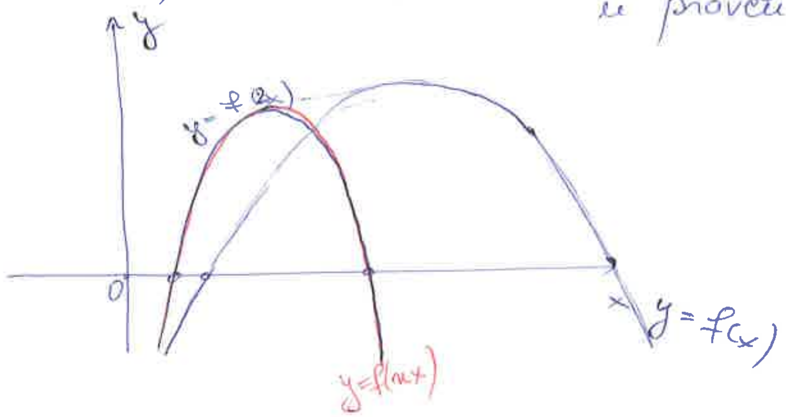
VI slučaj Nacrtati sliku krive $y=\frac{f(x)}{m}$ ako se zna slika krive $y=f(x)$.

Primjer: $y=\frac{1}{2}\sin x$



VII slučaj $y=f(mx)$ $(m>1)$ – „kontrakcija" krive $y=f(x)$ u pravcu ose Ox

$m=2$



Primjer:

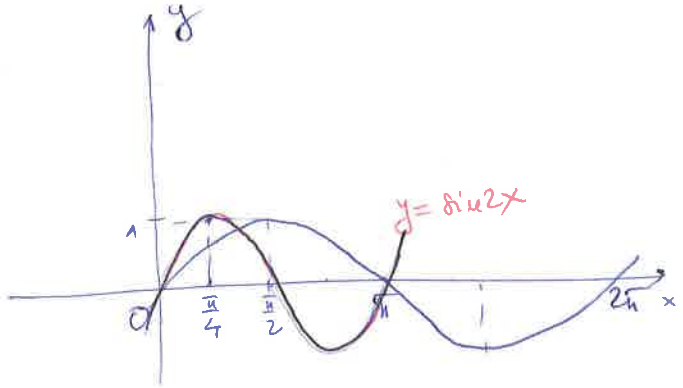
$y=\sin(2x)$

$y=\sin 2x$

$\sin 0 = 0$

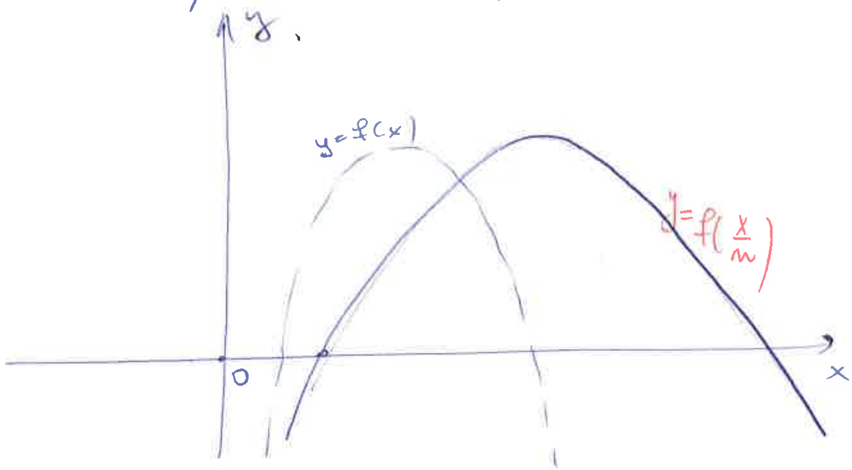
$\sin 2\cdot\frac{\pi}{2} = \sin\pi = 0$

$\sin 2\cdot\frac{\pi}{4} = 1$



VIII slučaj $y=f\left(\frac{x}{m}\right)$ $(m>1)$ – dobijamo „rastezanjem" u pravcu ose Ox slike krive $y=f(x)$.

$m=2$



Primjer: $y = \sin \frac{x}{2}$